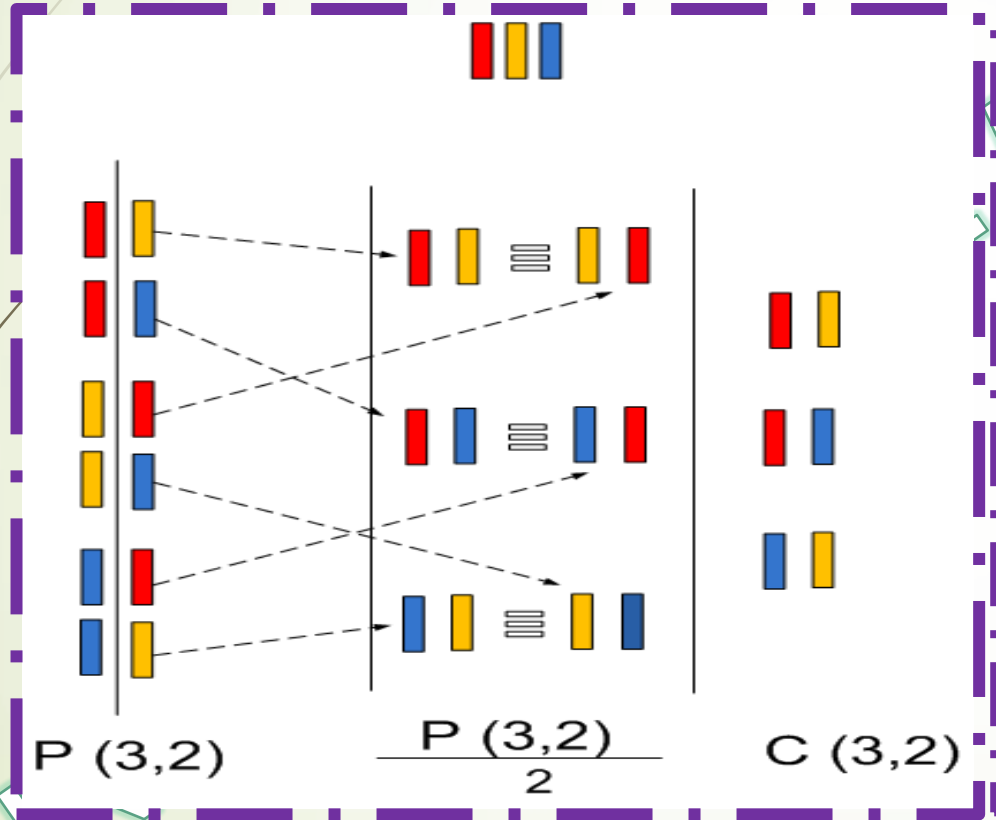
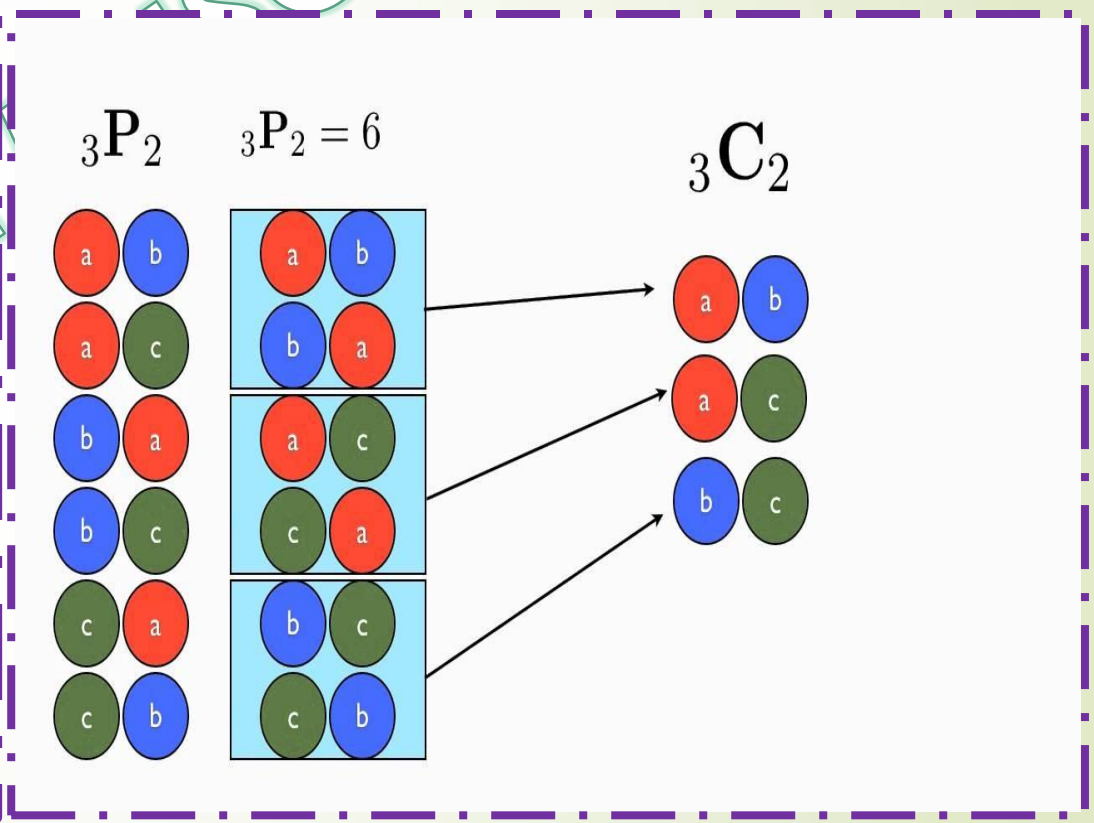
COMBINATIONS.....

MODULE-7





RECAP.....

Combinations

- Consider the six permutations of $\{p, e, n\}$ which are grouped in three pairs of 2. Each pair corresponds to one combination of 2:

$$(pe, ep), \quad (pn, np), \quad (en, ne)$$

- If we want to find the number of combinations of 3 objects taken 2 at a time, we simply divide the number of permutations of 3 objects taken 2 at a time by 2 (or 2!)
- We have the following result:

$$C(3, 2) = \frac{P(3, 2)}{2!}$$

Combinations

A **Combination** is an arrangement of items in which order does not matter & Repetition is NOT allowed.

To find the number of Combinations of n items chosen r at a time:

$$ {}_nC_r = C(n,r) = \frac{n!}{r!(n-r)!} $$

IMPORTANT RESULTS - RECAP.....

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{n!}{(n-r)! \times r!} \times r! = {}^{n}C_{r} \times r! $$

$ {}^{n}C_{n} = 1 $ and $ {}^{n}C_{0} = 1 $

$$ {}^{n}C_{r} = {}^{n}C_{n-r} $$

If $ {}^{n}C_{a} = {}^{n}C_{b} $ **Then, either** $a = b$ **or** $a + b = n$

QUESTIONS.....

Example If ${}^{n}C_9 = {}^{n}C_8$, find n

Solution We have ${}^{n}C_9 = {}^{n}C_8$

i.e., $$\frac{n!}{9!(n-9)!} = \frac{n!}{(n-8)!\,8!}$$

or $$\frac{1}{9} = \frac{1}{n-8} \quad \text{or} \quad n-8 = 9 \quad \text{or} \quad n = 17$$

If $nC_a = nC_b$, then either $a = b$ or $a + b = n$

If ${}^{n}C_9 = {}^{n}C_8$, find n.

Here, since a ≠ b, so,
n = a+b = 9 + 8 = 17.

Find r if $18C_r = 18\,C_{r+2}$

Here, a ≠ b, so, by using

If $nC_a = nC_b$, then either $a = b$ or $a + b = n$

$\Rightarrow$ r + r+2 = 18

$\Rightarrow$ 2 r = 16 $\Rightarrow$ r = 8

QUESTIONS…..

Q 1) In a society of 10 members, we have to select a committee of 4 members. As the owner of the society, John is already a member of the committee. In how many ways the committee can be formed.

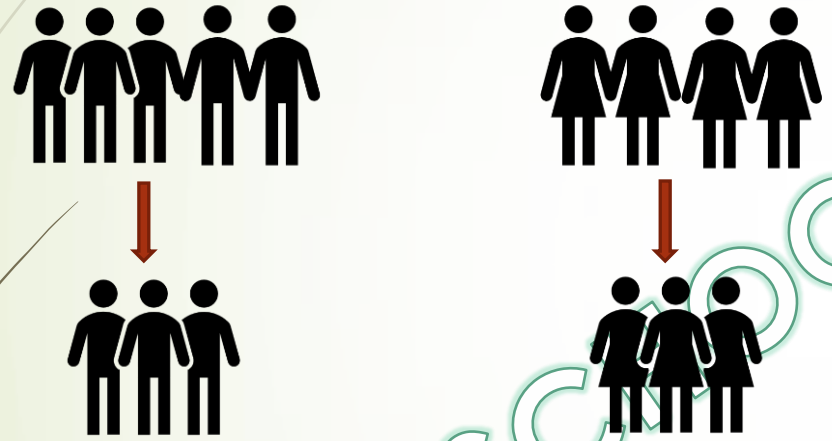
Solution: **John is already a part of the committee. Thus, we have to select 3 members among 9.**

We can select 3 members from 9 members in $9C_3$ ways

which is equal to $\frac{9!}{6!3!}$ = 84 ways

QUESTIONS…..

Q2) In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

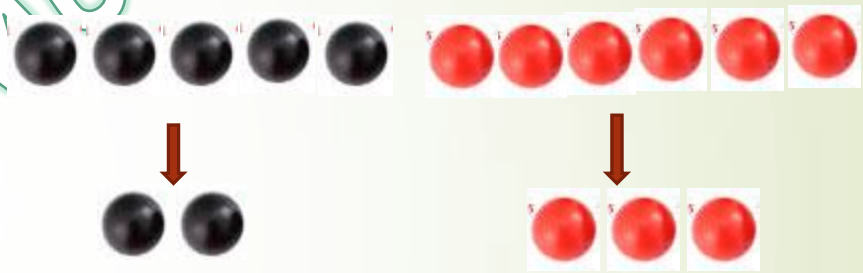


No. of selections = ${}^5C_3 \times {}^4C_3$

$= {}^5C_2 \times {}^4C_1 = \frac{5!}{2!3!} \times 4$

$= 10 \times 4 = 40$

Q3) A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.



No. of selections = ${}^5C_2 \times {}^6C_3$

$= \frac{5!}{2!3!} \times \frac{6!}{3!3!}$

$= 10 \times 20 = 200$

QUESTIONS…..

Q4) In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Total no. of players = 17

5 bowlers 12 non-bowlers

4 7

No. of selections = ${}^{5}C_{4} \times {}^{12}C_{7}$

$= \frac{5!}{4!} \times \frac{12!}{5!7!}$

= 5 x 792 = 3960

Q5) Determine 'n' if ${}^{2n}C_{3} : {}^{n}C_{2} = 12:1$

$\frac{(2n)!}{3!\,(2n-3)!} : \frac{n!}{2!\,(n-2)!} = 12:1$

$\Rightarrow \frac{2n(2n-1)(2n-2)(2n-3)!}{3\times 2!\,(2n-3)!} \times \frac{2!\,(n-2)!}{n!} = \frac{12}{1}$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{12}{1}$

$\Rightarrow \frac{4(2n-1)}{3} = \frac{12}{1}$

$\Rightarrow 8n - 4 = 36$

$\Rightarrow n = 5$

QUESTIONS……

Q6) Find the number of ways of selecting 9 balls from 6 red balls, 5 green balls and 5 blue balls if each selection consists of 3 balls of each colour.

Total number of balls = 6 R + 5 G + 5 B

3 3 3

No. of selections = $^{6}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{3}$

$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!3!}$

$= 20 \times 10 \times 10 = 2000$

7) In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Total number of courses = 9

2 compulsory → 2

remaining 7 → 3

No. of ways $= {}^{2}C_{2} \times {}^{7}C_{3} = 1 \times \frac{7!}{4!3!}$

$= 1 \times 35 = 35$

QUESTIONS…..

Q8) In how many ways can a football team of 11 players be selected from 16 players? How many of them will (i) include two particular players? (ii) exclude two particular players?

- No. of ways of selecting 11 players from 16 players = ${}^{16}C_{11} = \frac{16!}{11!5!} = 4368$
- (i) If two particular players are included :

16

2 included 14 (select 9)

No. of selections = ${}^{14}C_9 = \frac{14!}{9!5!} = 2002$

- (ii) If two particular players are excluded :

16

2 excluded 14 (select 11)

No. of selections = ${}^{14}C_{11} = \frac{14!}{3!11!} = 364$

THEOREM.....

Theorem – $${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$

Proof We have $${}^{n}C_{r} + {}^{n}C_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

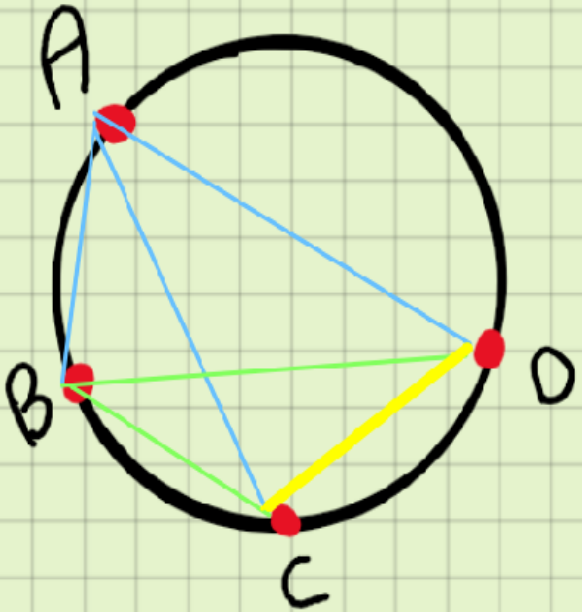
$$= \frac{n!}{r \times (r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

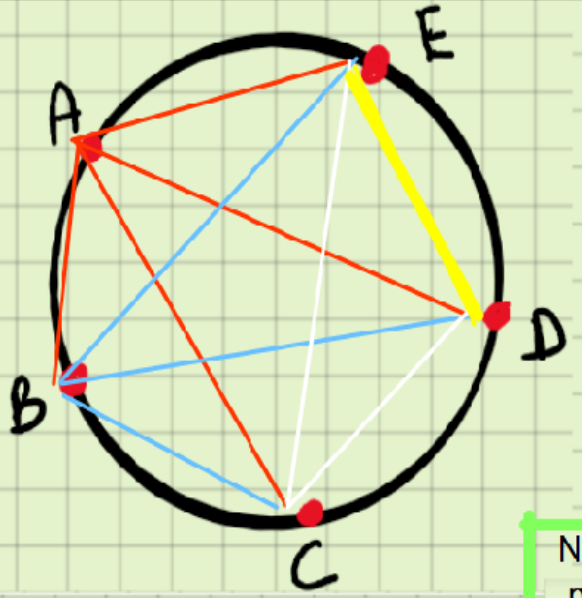
$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

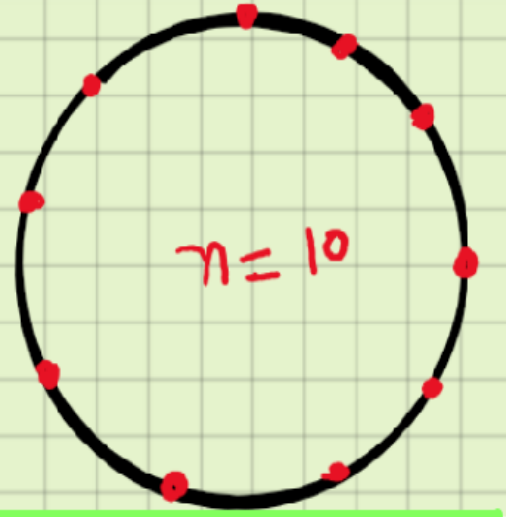
$$= \frac{n!}{(r-1)!(n-r)!} \times \frac{n-r+1+r}{r(n-r+1)} = \frac{(n+1)!}{r!(n+1-r)!} = {}^{n+1}C_{r}$$

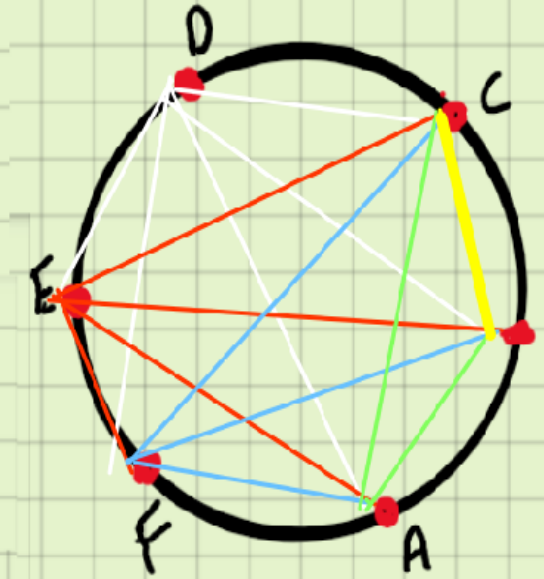
Number of lines that can be drawn through some points on the circle

1. WHY CIRCLE ?
2. WHY NOT PLANE ?









No. of points	Total lines	Deduction
4	6	3, 2, 1
5	10	4, 3, 2, 1
6	15	5, 4, 3, 2, 1
10	45	9, 8, 7, 6, 5, 4, 3, 2, 1

Can you deduce the result for n points

No. of lines that can be drawn .....

- When n=4, no. of lines drawn =3+2+1 = 6
- When n=5, no. of lines drawn

  = 4+3+2+1= 10
- When n=6, no. of lines drawn

  = 5+4+3+2+1 = 15
- When n=10. no. of lines drawn

  = 9+8+7+6+5+4+3+2+1= 45

So, In general, when there are 'n' points on a circle, no. of lines drawn

=(n-1)+(n-2)+(n-3)+…..+3+2+1

= sum of first (n-1) natural numbers

$= \frac{n(n-1)}{2} = {}^{n}C_{2}$

Q) How many chords can be drawn through 21 points on a circle?

Ans) There are 21 points on the circle. Since only one chord can be drawn by joining 2 distinct points, so the required no. of chords is:

$${}^{21}C_{2} = \frac{21!}{19!2!} = \frac{21 \times 20}{2} = 210$$

If there are 38 points ????

HOMEWORK QUESTIONS……

1) A bag contains 6 white marbles and 5 red marbles. Find the number of ways in which 4 marbles can be drawn from the bag if (a) they can be of any colour (b) two must be white and two red and (c) they must all be of the same colour.
2) In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.
3) From 3 officers and 8 jawans, in how many ways can 6 be chosen to (a) include one particular officer (b) exclude one particular officer.
4) How many different committees each consisting of 3 girls and 2 boys can be chosen from 7 girls and 5 boys?
5) In how many ways can a football team of eleven plyers be selected from 15 players? (i) In how many of them a particular player is included? (ii) In how many of them he is excluded?

ANSWERS :

Q.1) (a) 330 (b) 150 (c) 20 Q.2) 3 Q.3) (a) 252 (b) 210 Q.4) 350

Q.5) 1365 (i) 1001 (ii) 364